

The Schwarzschild/CFT Correspondence: Weyl Rescaled Case

Jafar Sadeghi ^{a*}, and Vahid Reza Shajiee ^{b†}

^a *Department of Physics, University of Mazandaran,
Babolsar, Iran.*

^b *Young Researchers and Elite Club, Mashhad Branch, Islamic Azad University,
Mashhad, Iran.*

Abstract

In this work, the CFT dual of the Schwarzschild black hole is investigated. A Weyl rescaling factor is presented, so that the Weyl rescaled Schwarzschild metric, after a coordinate transformation, has an $AdS_2 \times S_2$ geometry at vicinity of its origin. Since the near origin spacetime admits an AdS_2 factor, it is dealt with a 2D effective gravity which is dimensionally reduced from the near origin solution. It is exhibited that the dual CFT has a central charge $c = 96M^3$ which is an asymptotic conserved charge of the effective solution. Finally, the microscopic entropy of Schwarzschild black hole is achieved by using Cardy formula. It is revealed that the microscopic entropy exactly reproduces the Bekenstein-Hawking entropy of the Schwarzschild black hole.

Keywords: AdS/CFT Correspondence, Schwarzschild Black Hole, Quantum Gravity, Central Charge

1 Introduction

It is many years that physicists have been searching for quantum gravity. Although the problem is still open, promising works have been done in this issue [1, 2, 3, 4]. One of the most influential works is Maldacena's conjecture on AdS/CFT [5]. However, it does not directly lead to quantum gravity, it seems like a key for future works. On the one side, there exists a (quantum) gravity framework, and on the other side, there exists a conformal field theory framework. The conjecture was first proposed for superstring theory, although it was extended later to other issues in physics [6, 7, 8, 9].

Certainly, the physics in low dimensions is much more soluble and easier than high dimensions case. For example, because 1+1 dimensions pure gravity is trivial, there exist no propagating modes, there are some hopes to find quantum descriptions [10, 11,

*Email: pouriya@ipm.ir

†Email: v.shajiee@mshdiau.ac.ir

12]. Since 1+1 dimensional pure gravity is degenerate¹, one may add some auxiliary fields into the theory to stabilize degeneracy. So, in 1+1 dimensions one may need more than Einstein theory of gravity. A proposed theory for 1+1 dimensions gravity is 2D Maxwell-dilaton gravity, which dilaton part arises in Polyakov string theory [13, 14].

According to what has been stated, a question then arises as how one can examine quantum gravity. An answer might be black holes, because black holes live in the background spacetime of (quantum) gravity, and also they have well-known properties which might be measurable in quantum gravity. As it has been known, a black hole possesses an anti-de Sitter geometry in its near horizon [15]. So, according to AdS/CFT, it is natural to search out a dual CFT for this AdS spacetime. In this regard, a great investigation was presented as the Kerr/CFT correspondence [16], which could be generally called Near Horizon/CFT². Later, such type of investigation was extended to another black holes [18, 19, 20, 21, 22].

Considering Noether's theorem corresponding to every continuous symmetry, there exists a conserved charge. So, there might be a conserved charge corresponding to asymptotic symmetries of the spacetime. In the first place, Brown and Henneaux had shown the relationship between the conserved charges and a nontrivial central extension of the asymptotic symmetry algebra in AdS_3 spacetime [23]. In the recent years, it is known that the nontrivial asymptotic symmetry generators at spatial infinity for AdS spacetimes are in one-to-one correspondence to the conformal symmetry generators of the boundary [24].

Between black hole solutions, the Schwarzschild black hole is so quiet. It represents an excited quantum system which is decaying [25]. The Schwarzschild black hole is only described by its mass, this would probably be the reason why it has not been investigated successfully in concept of holographic duality, so far [16]. In [26], the Schwarzschild black hole has been investigated in concept of AdS_2/CFT_1 via RN/CFT correspondence.

In this paper, the Schwarzschild black hole, after a Weyl rescaling, is investigated from AdS_2/CFT_1 correspondence point of view. In section 2, the near origin geometry of the Weyl rescaled coordinate transformed Schwarzschild black hole is discussed. In section 3, following [27, 18], the 2D effective action of 4D Schwarzschild black hole is obtained via dimensional reduction. In section 4, the boundary stress tensor is calculated. In section 5, the dual CFT for the Schwarzschild black hole is investigated. In section 6, some conclusions are demonstrated.

2 Near origin geometry

Regular Schwarzschild black hole is a static spherically symmetric solution to the Einstein-Hilbert action in four dimensions,

$$I = \frac{1}{16\pi G_4} \int d^4x \sqrt{-g_4} R. \quad (1)$$

¹See [13].

²A good review is presented in [17].

The metric is given by

$$ds^2 = -(1 - \frac{2M}{r})dt^2 + (1 - \frac{2M}{r})^{-1}dr^2 + r^2d\Omega^2 \quad (2)$$

where M is the mass. The event horizon, the Hawking temperature and the entropy of the black hole are given, respectively, by

$$r_h = 2M \quad (3)$$

$$T_H = \frac{\kappa}{2\pi} = \frac{1}{8\pi M} \quad (4)$$

$$S_{B.H} = \frac{A}{4} = 4\pi M^2 \quad (5)$$

where κ and A are the surface gravity and the surface area of the horizon, respectively.

It is known that black hole thermodynamics are Weyl invariance [28, and references therein]. This statement may lead one to study conformal cousins of regular black holes. For the Schwarzschild black hole, considering the Weyl rescaling of the metric,

$$g_{\mu\nu} \rightarrow \Omega^{-2}g_{\mu\nu}, \quad (6)$$

where the Weyl rescaling factor Ω is introduced as follows

$$\Omega^{-2} = \frac{4M^2}{r^2}, \quad (7)$$

the Weyl rescaled Schwarzschild metric, after coordinate transformation $r \rightarrow \frac{1}{r}$, would be obtained as follows

$$ds^2 = -4M^2(r^2 - 2Mr^3)dt^2 + \frac{4M^2}{r^2 - 2Mr^3}dr^2 + 4M^2d\Omega^2. \quad (8)$$

Penrose diagrams of the exterior region of the Schwarzschild black hole (2), i.e. $2M \leq r \leq \infty$, and the interior region of the case (8), i.e. $0 \leq r \leq \frac{1}{2M}$, are the same [28]. This is a good motivation to consider the interior region of (8), $0 \leq r \leq \frac{1}{2M}$, in the following. Also, at least so far, it is well known that the dual CFT information could be read off from outside of the horizon, this is another motivation.

The Weyl rescaled coordinate transformed Schwarzschild metric, the equation (8) where $0 \leq r \leq \frac{1}{2M}$, has an $AdS_2 \times S_2$ geometry at vicinity of its origin. Taking the following limit

$$t \rightarrow \frac{t}{4M^2\epsilon}, \quad r \rightarrow 0 + \epsilon r, \quad \epsilon \rightarrow 0, \quad (9)$$

one could easily get the near origin metric,

$$ds^2 = -\frac{r^2}{4M^2}dt^2 + \frac{4M^2}{r^2}dr^2 + 4M^2d\Omega^2, \quad (10)$$

where $0 \leq r \leq \infty$. The equation (10) describes an $AdS_2 \times S_2$ geometry. The AdS_2 radius and the 2-sphere radius are $\ell_{AdS_2} = 2M$ and $\ell_{S_2} = 2M$, respectively. The Ricci scalars are as follows

$$R_{Near\ Origin} = R_{AdS_2} + R_{S_2} = 0, \quad (11)$$

$$R_{AdS_2} = -\frac{1}{2M^2}. \quad (12)$$

The near origin spacetime is Ricci flat.

3 2D effective theory

To search for 2D effective theory, let us introduce the following map

$$r \mapsto e^{\frac{r}{2M}}. \quad (13)$$

Putting the map (13) into the metric (10), one could obtain

$$ds^2 = -\frac{e^{\frac{r}{M}}}{4M^2} dt^2 + dr^2 + 4M^2 d\Omega^2, \quad (14)$$

In this metric, AdS_2 part is the Fefferman-Graham type. It describes the near boundary of the near origin metric. The Ricci scalar is the same.

By reduction the 4D Einstein gravity to the 2D Einstein gravity, the 2D action could be obtained; while as expressed in section 1, the 2D pure gravity results degenerate equations of motion, instead of dynamical equations [29, 13]. Here is where one could put some auxiliary field into the theory to get dynamical equations of motion. The 2D Maxwell-dilaton gravity is known as a well-defined 2D theory of gravity, because of this, it is chosen in this work. The action of the 2D Maxwell-dilaton gravity is given by

$$I = \alpha \int d^2x \sqrt{-g_{AdS_2}} (e^{-2\Phi} (R_{AdS_2} + \frac{8}{L^2}) - \frac{L^2}{4} F^2) \quad (15)$$

where $L = 2\ell_{AdS}$ and α is a normalization constant [30, and references therein]. After reducing the action from four dimensions to two dimensions, adding a dilaton field³ and a minimally coupled U(1) gauge field to the reduced action, the 2D effective action could be written as follows

$$I = \frac{M^2}{G_4} \int d^2x \sqrt{-g_{AdS_2}} (e^{-2\Phi} (R_{AdS_2} + \frac{1}{2M^2}) - F^2), \quad (16)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the U(1) gauge field strength. The corresponding equations of motion for constant Φ are

$$R_{AdS_2} + \frac{1}{2M^2} = 0, \quad (17)$$

$$\nabla_\mu F^{\mu\nu} = 0, \quad (18)$$

$$\frac{1}{2} g_{\mu\nu} (\frac{1}{4M^2} e^{-2\Phi} - \frac{1}{2} F^2) + F_{\mu\alpha} F_\nu^\alpha = 0. \quad (19)$$

As it is mentioned in [18], constant Φ is a consistent truncation. The AdS_2 part of the metric (14) could be a solution to the equations of motion, if one takes the following ansatz

$$A = A_t(t, r) dt. \quad (20)$$

So, putting the AdS_2 part of the metric (14) and the ansatz (20) into the equations of motion, one could get

$$A = \frac{1}{2M} e^{\frac{r}{2M} - \Phi} dt. \quad (21)$$

³The dilaton field Φ would be zero, however it must be held to avoid degeneracy.

4 Boundary stress tensor

When spacetime has a boundary, some additional terms defined on the boundary should be added to the bulk action

$$I = I_{bulk} + I_{boundary}. \quad (22)$$

The boundary action includes the Gibbons-Hawking-York boundary term and the counterterms

$$I_{boundary} = I_{G.H.Y} + I_{counter}. \quad (23)$$

In the present work, $I_{G.H.Y}$ and $I_{counter}$ are given by

$$I_{G.H.Y} = \frac{2M^2}{G_4} \int dt \sqrt{-\gamma} e^{-2\Phi} K, \quad (24)$$

$$I_{counter} = \frac{2M^2}{G_4} \int dt \sqrt{-\gamma} (\alpha e^{-2\Phi} + \beta A_t A^t), \quad (25)$$

where $\gamma_{tt} = g_{tt}$ is the time-like boundary metric and α, β are the constraint coefficients which should be calculated. The extrinsic curvature is given by

$$K = \frac{1}{2} \gamma^{tt} n^\mu \partial_\mu \gamma_{tt} = \frac{1}{2M}, \quad (26)$$

here n^μ is the space-like unit vector normal to the boundary. Varying the full action leads to

$$\delta I = \int dt \sqrt{-\gamma} (\pi_{tt} \delta \gamma^{tt} + \pi_\Phi \delta \Phi + \pi^t \delta A_t) + (bulk\ terms), \quad (27)$$

where

$$\pi_{tt} = -\frac{M^2}{G_4} (\alpha e^{-2\Phi} \gamma_{tt} + \beta \gamma_{tt} A_t A^t - 2\beta A_t A_t) = -\frac{M^2}{G_4} e^{-2\Phi} \gamma_{tt} (\alpha + \beta), \quad (28)$$

$$\pi_\Phi = -\frac{2M^2}{G_4} (2e^{-2\Phi} K + 2\alpha e^{-2\Phi}) = -\frac{4M^2}{G_4} e^{-2\Phi} \left(\frac{1}{2M} + \alpha \right), \quad (29)$$

$$\pi^t = \frac{M^2}{G_4} (-4n_\mu F^{\mu t} + 4\beta A^t) = \frac{8M^2}{G_4} e^{-\Phi} e^{-\frac{r}{2M}} (1 - 2M\beta). \quad (30)$$

These equations are finite which show that the good counterterms have been chosen. As strong constraints, the above momenta could be set to zero, hence the constraint coefficients could be obtained as follows

$$\alpha = -\frac{1}{2M}, \quad \beta = \frac{1}{2M}. \quad (31)$$

The boundary stress tensor is defined by

$$T_{ab} = -\frac{2}{\sqrt{-\gamma}} \frac{\delta I}{\delta \gamma^{ab}} = -2\pi_{ab}, \quad (32)$$

and so its corresponding component would be

$$T_{tt} = -\frac{M}{G_4} (e^{-2\Phi} \gamma_{tt} + A_t A_t) = 0. \quad (33)$$

5 The dual CFT

To get the conserved charge associated with central charge, some methods was proposed in [23, 31, 32]. One of them is investigation of the action of the bulk diffeomorphism on the boundary stress tensor which works well even in low dimensions [33, 28, 30, 34].

To find the central charge, one could start with finding the Killing vectors of AdS_2 part of the near origin spacetime

$$ds^2 = -\frac{e^{\frac{r}{M}}}{4M^2} dt^2 + dr^2. \quad (34)$$

The Killing vectors would be the generators of bulk diffeomorphism group. Solving Killing equation

$$\mathcal{L}_\xi g = 0 \quad (35)$$

in the background (34), the Killing vectors could be obtained as follows

$$\xi_{+1} = -(16M^4 e^{-\frac{r}{M}} + t^2) \partial_t + 4Mt \partial_r, \quad (36)$$

$$\xi_0 = -t \partial_t + 2M \partial_r, \quad (37)$$

$$\xi_{-1} = -\partial_t. \quad (38)$$

the Killing vectors form $SL(2, R)$ algebra. Also, they include the $SO(1, 2) \sim SL(2, R)$ algebra on the 1D timelike background at boundary. Equivalently, they could be rewritten as

$$\xi = -(8M^4 e^{-\frac{r}{M}} \ddot{\zeta}(t) + \zeta(t)) \partial_t + (2M \dot{\zeta}(t)) \partial_r, \quad (39)$$

where ‘dot’ denotes derivative with respect to ‘t’ and $\zeta(t)$ is a smooth function of time. The equation (39) is a good choice as the asymptotic generator, because of two reason, first it transforms the background metric (34) as

$$\mathcal{L}_\xi g = 4M^2 \ddot{\zeta}(t) dt^2, \quad (40)$$

which is a strong boundary condition (another components are zero), second the equation (39) does not diverge at boundary. It should be mentioned that there also exists the $U(1)$ gauge field (21) and it should be investigated how it transforms. Without difficulty, one could obtain

$$\mathcal{L}_\xi A = -4M^3 e^{-\frac{r}{2M} - \Phi} \ddot{\zeta}(t) dt + 4M^2 e^{-\frac{r}{2M} - \Phi} \ddot{\zeta}(t) dr. \quad (41)$$

Obviously, the second term violates the gauge condition $A_r = 0$. The violation could be handled by the $U(1)$ gauge fixing $A_\mu \rightarrow A_\mu + \partial_\mu \Lambda$, this leads to

$$\Lambda = 8M^3 e^{-\frac{r}{2M} - \Phi} \ddot{\zeta}(t). \quad (42)$$

By the gauge fixing, one could rewrite the compensated gauge transformation as

$$(\delta_{\xi+\Lambda} A)_\mu = (\mathcal{L}_\xi A)_\mu + \partial_\mu \Lambda, \quad (43)$$

its corresponding component is

$$(\delta_{\xi+\Lambda} A)_t = (\mathcal{L}_\xi A)_t + \partial_t \Lambda = 4M^3 e^{-\frac{r}{2M} - \Phi} \ddot{\zeta}(t). \quad (44)$$

The action of the bulk diffeomorphism ξ on the boundary stress tensor T_{tt} could be caculated as

$$\begin{aligned}\delta_{\xi+\Lambda}T_{tt} &= -\frac{M}{G_4}(e^{-2\Phi}(\mathcal{L}_\xi g)_{tt} + 2A_t(\delta_{\xi+\Lambda}A)_t) \\ &= -\frac{8M^3}{G_4}e^{-2\Phi}\ddot{\zeta}(t).\end{aligned}\tag{45}$$

The central charge could be read off from the following relation,

$$\delta_{\xi+\Lambda}T_{tt} = 2T_{tt}\dot{\zeta}(t) + \zeta(t)\dot{T}_{tt} - \frac{c}{12}\ddot{\zeta}(t).\tag{46}$$

Comparing equations (45),(46) and taking $\Phi = 0$, the central charge is obtained in the natural units $G_4 = 1^4$

$$c = 96M^3.\tag{47}$$

The microscopic entropy could be achieved by using Cardy formula,

$$S_{micro} = \frac{\pi^2}{3}cT_{CFT}.\tag{48}$$

It is expected that in a chiral CFT the chiral temperature would be equal to the Hawking temperature. So, taking $T_{CFT} = \frac{1}{8\pi M} = T_H$, the microscopic entropy of the Schwarzschild black hole is obtained as

$$S_{micro} = 4\pi M^2 = S_{macro}.\tag{49}$$

As expected, it reproduces the macroscopic Bekenstein-Hawking entropy (5).

6 Conclusions

In this letter, the Schwarzschild black hole has been investigated from AdS_2/CFT_1 correspondence point of view. Since black hole thermodynamics are Weyl invariance, the Weyl rescaled Schwarzschild black hole has been considered. It has been shown that the Weyl recaled coordinate transformed Schwarzschild black hole has a well-defined $AdS_2 \times S_2$ geometry at vicinity of its origin. Corresponding to the near origin metric, the 2D action has been procured from 4D Einstein-Hilbert action via dimensional reduction. Since the 2D pure gravity is degenerate, some auxiliary fields has been added to get a 2D effective theory which leads to dynamical equations of motion. Note that the equation (17) has been obtained from variation of the action with respect to the dilaton field; therefore, the dilaton field should be included to avoid degeneracy. Obtaining the boundary stress tensor and interrogating the action of asymptotic diffeomorphism of the bulk on the boundary stress tensor, the central charge of the dual CFT has been calculated. It has been shown that the microscopic entropy reproduces the macroscopic entropy of the Schwarzschild black hole.

⁴One might put a normalize factor L into the equation (46) to get the dimensionless central charge [30, 18]. Note that in that case, one should define the dimensionless Hawking temperature. Anyway, both cases lead to the same results.

One might apply the 2D dilaton gravity without $U(1)$ gauge field. It is expected that the results would be the same. It should be noticed that in that case the dilaton field could not anymore be constant. Also, as an open problem, a supersymmetric gauge field might be tackled to the problem, instead of the $U(1)$ gauge field. Such investigation should lead to the same result, nevertheless it will have interesting mathematical and physical points.

References

- [1] G. 't Hooft, *Dimensional reduction in quantum gravity*, in *Salamfest 1993:0284-296*, pp. 0284–296, 1993. [gr-qc/9310026](#).
- [2] L. Susskind, *The World as a hologram*, *J. Math. Phys.* **36** (1995) 6377–6396, [[hep-th/9409089](#)].
- [3] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253–291, [[hep-th/9802150](#)].
- [4] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, *Gauge theory correlators from noncritical string theory*, *Phys. Lett.* **B428** (1998) 105–114, [[hep-th/9802109](#)].
- [5] J. M. Maldacena, *The Large N limit of superconformal field theories and supergravity*, *Int. J. Theor. Phys.* **38** (1999) 1113–1133, [[hep-th/9711200](#)]. [*Adv. Theor. Math. Phys.* 2,231(1998)].
- [6] S. A. Hartnoll, C. P. Herzog, and G. T. Horowitz, *Building a Holographic Superconductor*, *Phys. Rev. Lett.* **101** (2008) 031601, [[arXiv:0803.3295](#)].
- [7] S. Sachdev, *Condensed Matter and AdS/CFT*, [arXiv:1002.2947](#). [*Lect. Notes Phys.* 828,273(2011)].
- [8] D. T. Son, *Toward an AdS/cold atoms correspondence: A Geometric realization of the Schrodinger symmetry*, *Phys. Rev.* **D78** (2008) 046003, [[arXiv:0804.3972](#)].
- [9] C. P. Herzog, A. Karch, P. Kovtun, C. Kozcaz, and L. G. Yaffe, *Energy loss of a heavy quark moving through $N=4$ supersymmetric Yang-Mills plasma*, *JHEP* **07** (2006) 013, [[hep-th/0605158](#)].
- [10] F. I. Cooperstock and V. Faraoni, *Lower dimensional theories of gravity and gravitational waves*, *Gen. Rel. Grav.* **27** (1995) 555–562.
- [11] E. Witten, *Three-Dimensional Gravity Revisited*, [arXiv:0706.3359](#).
- [12] P. H. Ginsparg and G. W. Moore, *Lectures on 2-D gravity and 2-D string theory*, in *Yale Univ. New Haven - YCTP-P23-92 (92,rec.Apr.93) 197 p. Los Alamos Nat. Lab. - LA-UR-92-3479 (92,rec.Apr.93) 197 p. e: LANL hep-th/9304011, In *Boulder 1992, Proceedings, Recent directions in particle theory* 277-469. and Yale Univ. New Haven - YCTP-P23-92 (92,rec.Apr.93) 197 p. and Los Alamos Nat. Lab. - LA-UR-92-3479 (92,rec.Apr.93) 197 p, 1993. [hep-th/9304011](#).*
- [13] G. Chechelashvili and G. Jorjadze, *Dilaton field and massless particle for 2-D gravity*, in *Mathematical methods in modern theoretical physics. Proceedings, International Meeting, School and Workshop, ISPM'98, Tbilisi, Georgia, September 5-18, 1998*, 1998. [hep-th/9908206](#).

- [14] A. M. Polyakov, *Quantum Geometry of Bosonic Strings*, *Phys. Lett.* **B103** (1981) 207–210.
- [15] H. K. Kunduri, J. Lucietti, and H. S. Reall, *Near-horizon symmetries of extremal black holes*, *Class. Quant. Grav.* **24** (2007) 4169–4190, [[arXiv:0705.4214](#)].
- [16] M. Guica, T. Hartman, W. Song, and A. Strominger, *The Kerr/CFT Correspondence*, *Phys. Rev.* **D80** (2009) 124008, [[arXiv:0809.4266](#)].
- [17] G. Compere, *The Kerr/CFT correspondence and its extensions: a comprehensive review*, *Living Rev. Rel.* **15** (2012) 11, [[arXiv:1203.3561](#)].
- [18] C.-M. Chen, J.-R. Sun, and S.-J. Zou, *The RN/CFT Correspondence Revisited*, *JHEP* **01** (2010) 057, [[arXiv:0910.2076](#)].
- [19] M. R. Garousi and A. Ghodsi, *The RN/CFT Correspondence*, *Phys. Lett.* **B687** (2010) 79–83, [[arXiv:0902.4387](#)].
- [20] J.-J. Peng and S.-Q. Wu, *Extremal Kerr/CFT correspondence of five-dimensional rotating (charged) black holes with squashed horizons*, *Nucl. Phys.* **B828** (2010) 273–288, [[arXiv:0911.5070](#)].
- [21] R. Li, M.-F. Li, and J.-R. Ren, *Entropy of Kaluza-Klein Black Hole from Kerr/CFT Correspondence*, *Phys. Lett.* **B691** (2010) 249–253, [[arXiv:1004.5335](#)].
- [22] A. M. Ghezelbash, *Kerr-Bolt Spacetimes and Kerr/CFT Correspondence*, *Mod. Phys. Lett.* **A27** (2012) 1250046, [[arXiv:0902.4662](#)].
- [23] J. D. Brown and M. Henneaux, *Central Charges in the Canonical Realization of Asymptotic Symmetries: An Example from Three-Dimensional Gravity*, *Commun. Math. Phys.* **104** (1986) 207–226.
- [24] G. Barnich, F. Brandt, and K. Claes, *Asymptotically anti-de Sitter space-times: Symmetries and conservation laws revisited*, *Nucl. Phys. Proc. Suppl.* **127** (2004) 114–117, [[gr-qc/0306112](#)]. [[,114\(2003\)](#)].
- [25] A. Strominger, *Five Problems in Quantum Gravity*, *Nucl. Phys. Proc. Suppl.* **192-193** (2009) 119–125, [[arXiv:0906.1313](#)].
- [26] V. R. Shajiee, *The Schwarzschild/CFT Correspondence*, *Int. J. Theor. Phys.* **55** (2016), no. 5 2376–2385.
- [27] A. Castro and F. Larsen, *Near Extremal Kerr Entropy from AdS(2) Quantum Gravity*, *JHEP* **12** (2009) 037, [[arXiv:0908.1121](#)].
- [28] M. Cadoni, *Statistical entropy of the Schwarzschild black hole*, *Mod. Phys. Lett.* **A21** (2006) 1879–1888, [[hep-th/0511103](#)].
- [29] H. Erbin, *Notes on 2d quantum gravity and liouville theory*, 2015.
- [30] A. Castro, D. Grumiller, F. Larsen, and R. McNees, *Holographic Description of AdS(2) Black Holes*, *JHEP* **11** (2008) 052, [[arXiv:0809.4264](#)].
- [31] J. D. Brown and J. W. York, Jr., *Quasilocal energy and conserved charges derived from the gravitational action*, *Phys. Rev.* **D47** (1993) 1407–1419, [[gr-qc/9209012](#)].

- [32] G. Barnich and F. Brandt, *Covariant theory of asymptotic symmetries, conservation laws and central charges*, *Nucl. Phys.* **B633** (2002) 3–82, [[hep-th/0111246](#)].
- [33] T. Hartman and A. Strominger, *Central Charge for $AdS(2)$ Quantum Gravity*, *JHEP* **04** (2009) 026, [[arXiv:0803.3621](#)].
- [34] S. Fischetti, W. Kelly, and D. Marolf, *Conserved Charges in Asymptotically (Locally) AdS Spacetimes*, [arXiv:1211.6347](#).